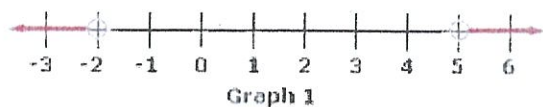


Name: Solutions / Answers

1. Write the domain of each graph in both **inequality** and **interval** notation.

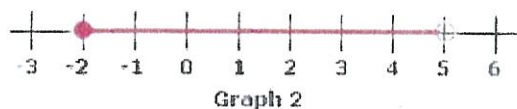


Inequality:

$$x < -2 \text{ or } x > 5$$

Interval:

$$(-\infty, -2) \cup (5, \infty)$$

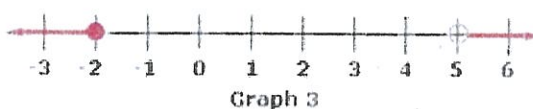


Inequality:

$$-2 \leq x < 5$$

Interval:

$$[-2, 5)$$

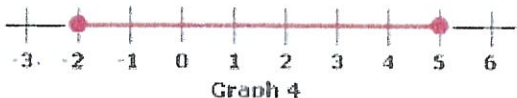


Inequality:

$$x \leq -2 \text{ or } x > 5$$

Interval:

$$(-\infty, -2] \cup (5, \infty)$$



Inequality:

$$-2 \leq x \leq 5$$

Interval:

$$[-2, 5]$$

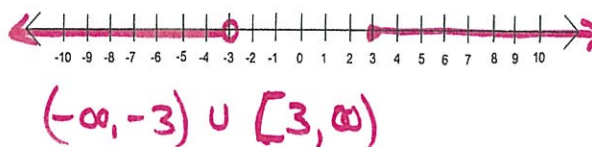
2. Graph the $(-\infty, 0] \cup (1, \infty)$ interval on a number line



3. Solve each inequality. Graph the solution set on a number line, AND write the solution in interval notation.

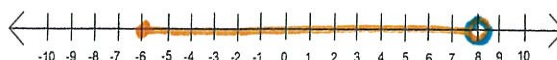
a. $3 - 2x > 9$ or $3x - 4 \geq 5$

$$\begin{aligned} -2x &> 6 & 3x &\geq 9 \\ x &< -3 & x &\geq 3 \end{aligned}$$



b. $-4 \leq \frac{1}{2}p - 1 < 3$

$$\begin{aligned} &+1 \quad +1 \quad +1 \\ 2(-3 &\leq \frac{1}{2}p < 4) \\ -6 &\leq p < 8 \end{aligned}$$



$$[-6, 8)$$

4. Write the inequalities $x < -6$ or $-1 \leq x < 6$ in interval notation.

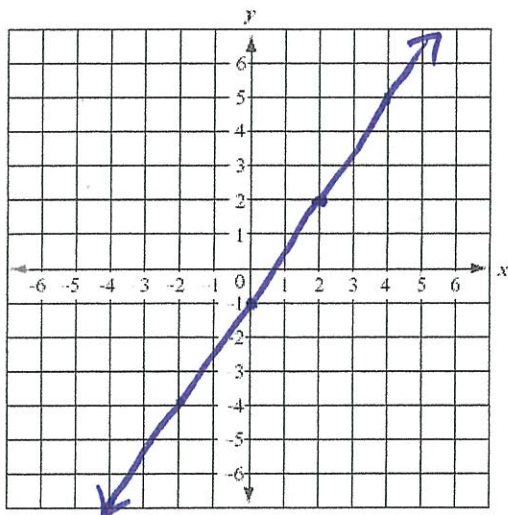
$$(-\infty, -6) \cup [-1, 6)$$

5. A rental car charge is \$100 per day plus \$0.20 per mile driven. Write an equation of the linear function that gives the total rental fee as a function of the number of miles driven in one day.

Let $y =$ rental fee Let $x =$ # of miles driven

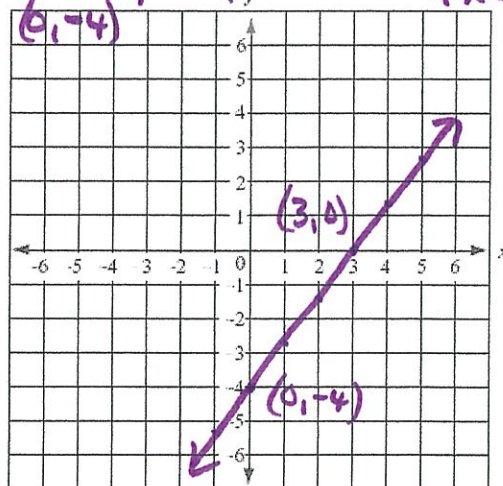
$$y = 0.20x + 100$$

6. Graph the line with equation $y = \frac{3}{2}x - 1$



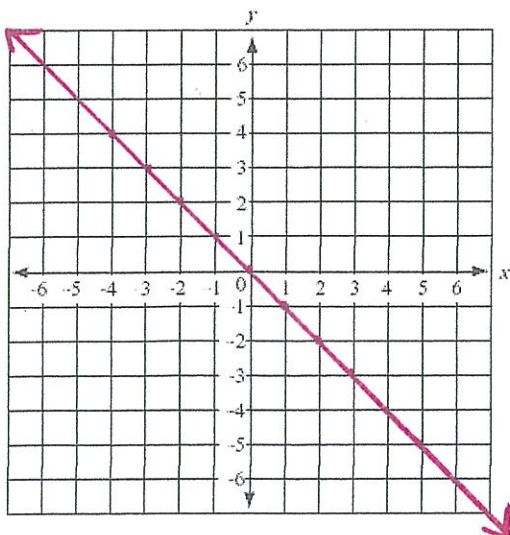
7. Graph the line with equation $4x - 3y = 12$

$$\begin{aligned} 4(0) - 3y &= 12 & 4x - 3(0) &= 12 \\ y &= -4 & 4x &= 12 & x &= 3 \end{aligned}$$

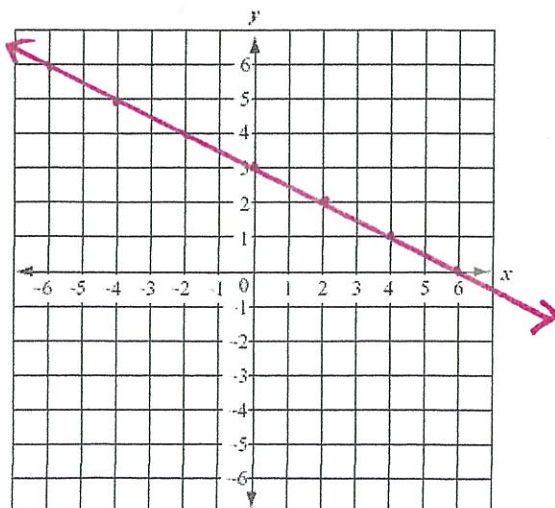


Given the lines shown, write an equation for each line in any form.

8. $y = -x$



9. $y = -\frac{1}{2}x + 3$



10. Given a line containing the points $(-4, 2)$ and $(8, 5)$. Find an equation of the line in slope-intercept form. Convert your equation to standard form.

$$m = \frac{5-2}{8-(-4)} = \frac{3}{12} = \frac{1}{4}$$

$$y = \frac{1}{4}x + b$$

$(8, 5)$ $5 = \frac{1}{4}(8) + b$

$$5 = 2 + b \quad b = 3$$

$$y = \frac{1}{4}x + 3$$

$$4y = x + 12$$

$$4y - x = 12$$

$$-x + 4y = 12$$

11. Line A has equation $y = \frac{-5}{3}x + 4$. Line B contains the point $(5, -4)$ and is perpendicular to line

A. Determine an equation for line B.

$$\text{line B: } y = \frac{3}{5}x + b$$

$$(5, -4) \quad -4 = \frac{3}{5}(5) + b$$

$$-4 = 3 + b$$

$$-7 = b$$

$$y = \frac{3}{5}x - 7$$

For #'s 12-15, simplify each expression (do not leave any negative exponents):

12. $81^{\frac{1}{4}} = 3$

13. $27^{\frac{4}{3}} = (27^{\frac{1}{3}})^4 = (3)^4 = 81$

14. $\frac{y^2}{y^{-1}} = y^2 \cdot y = y^3$

15. $(5x^2y^4)^2 - (2xy^2)^4 = 25x^4y^8 - 16x^4y^8 = 9x^4y^8$

$$\begin{aligned}
 16. \text{ Multiply } (x-7)^2 &= (x-7)(x-7) \\
 &= x^2 - 7x - 7x + 49 \\
 &= x^2 - 14x + 49
 \end{aligned}$$

$$\begin{aligned}
 17. \text{ Multiply } (2x+5)(4x-3) &= 8x^2 - 6x + 20x - 15 \\
 &= 8x^2 + 14x - 15
 \end{aligned}$$

$$18. \text{ Factor } m^2 - 7m + 10 = (m-2)(m-5)$$

$$19. \text{ Factor } 3x^2 - 5x - 12 = (3x + 4)(x - 3)$$

$$\begin{aligned}
 20. \text{ Solve } 0 &= 2y^2 - 14y && 0 = 2y(y-7) \\
 \underline{\underline{=}} &&& \downarrow \quad \downarrow \\
 &&& 2y = 0 \quad y - 7 = 0 \\
 &&& y = 0 \quad y = 7 && \{0, 7\}
 \end{aligned}$$

$$21. \text{ Convert the quadratic equation } f(x) = x^2 + 8x + 11 \text{ to vertex form}$$

$$\begin{aligned}
 \text{axis } x &= \frac{-8}{2(1)} = -4 && f(-4) = (-4)^2 + 8(-4) + 11 = -5 \\
 V(-4, -5) &&& y = (x+4)^2 - 5
 \end{aligned}$$

22. Convert the quadratic function $f(x) = 2(x-1)^2 - 3$ to standard form

$$f(x) = 2(x-1)(x-1) - 3$$

$$f(x) = 2(x^2 - 2x + 1) - 3$$

$$f(x) = 2x^2 - 4x + 2 - 3$$

$$f(x) = 2x^2 - 4x - 1$$

23. Graph $g(x) = x^2 - 4x - 5$

$$g(x) = (x-5)(x+1)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=5 & x=-1 \\ (5,0) & (-1,0) \end{array}$$

$$\text{axis } x = \frac{5+(-1)}{2}$$

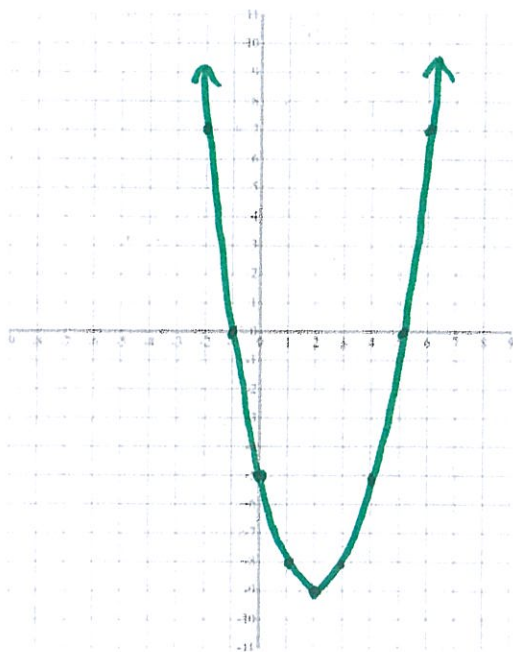
$$x = 2$$

$$g(2) = 2^2 - 4(2) - 5$$

$$g(2) = 4 - 8 - 5$$

$$g(2) = -9$$

$$V(2, -9)$$

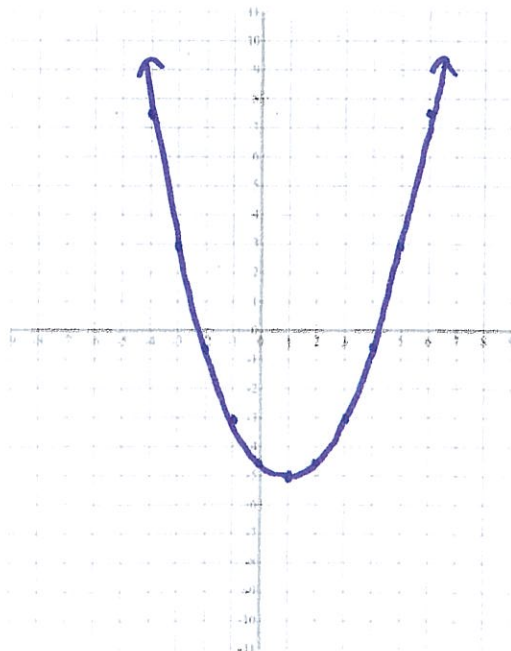


24. Graph $g(x) = \frac{1}{2}(x-1)^2 - 5$

$$V(1, -5)$$

pattern

$$\frac{1}{2} \quad 1\frac{1}{2} \quad 2\frac{1}{2} \quad 3\frac{1}{2} \quad 4\frac{1}{2}$$



25. Determine the real zeros (x-intercepts) of the parabola with equation $f(x) = \frac{1}{4}(x-3)^2 - 5$.

$$\begin{aligned}
 f(x) &= 0 \\
 0 &= \frac{1}{4}(x-3)^2 - 5 \\
 5 &= \frac{1}{4}(x-3)^2 \\
 20 &= (x-3)^2 \\
 \pm\sqrt{20} &= \sqrt{(x-3)^2}
 \end{aligned}$$

$$\begin{aligned}
 \pm 2\sqrt{5} &= x-3 \\
 3 \pm 2\sqrt{5} &= x \\
 \text{Zeros } x &= 3+2\sqrt{5} \text{ \& } x=3-2\sqrt{5} \\
 \text{x-intercepts } &(3+2\sqrt{5}, 0) \text{ \& } (3-2\sqrt{5}, 0)
 \end{aligned}$$

26. Determine the real zeros (x-intercepts) of the parabola with equation $g(x) = x^2 - 4x + 1$

quadratic Formula $a=1$ $b=-4$ $c=1$

$$\begin{aligned}
 x &= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} \\
 x &= \frac{4 \pm \sqrt{16-4}}{2} \\
 x &= \frac{4 \pm \sqrt{12}}{2}
 \end{aligned}$$

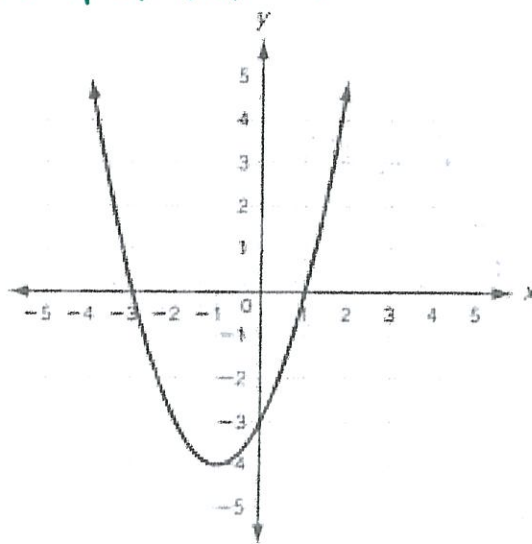
$$\begin{aligned}
 x &= \frac{4 \pm 2\sqrt{3}}{2} \\
 x &= 2 \pm \sqrt{3} \\
 \text{Zeros } x &= 2+\sqrt{3} \text{ \& } x=2-\sqrt{3} \\
 \text{x-intercepts } &(2+\sqrt{3}, 0) \text{ \& } (2-\sqrt{3}, 0)
 \end{aligned}$$

27. For the parabola shown, write a quadratic equation in the form of your choice.

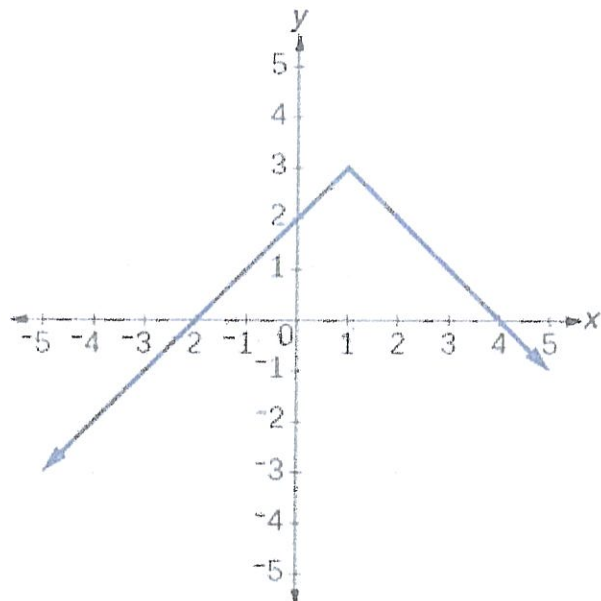
$V(-1, -4)$ Pattern 1, 3, 5, 7,

$$y = (x+1)^2 - 4$$

Vertex Form



Shown is the graph of $y = g(x)$



28. Evaluate:

a. $g(0) = 2$

b. $g(-4) = -2$

29. Write the domain and range of the function using interval notation.

$D: (-\infty, \infty)$ $R: (-\infty, 3]$

30. State the interval(s) on which the function is:

a. increasing

$(-\infty, 1)$

b. decreasing

$(1, \infty)$

31. State the interval(s) for which:

a. $g(x) > 0$

$-2 < x < 4$

b. $g(x) < 0$

$x < -2$ or $x > 4$

32. State each value:

a. the maximum value of $y = g(x)$

$y = 3$

b. the minimum value of $y = g(x)$

None

33. Solve $g(x) = -1$, i.e. for what value(s) of x does $g(x) = -1$ hold true?

$x = -3$ and $x = 5$

34. State the coordinates of each (approximate if necessary):

a. any x-intercepts

$(-2, 0)$ & $(4, 0)$

b. the y-intercept

$(0, 2)$

Use for #'s 35-39, given the two functions $f(x) = 2 - x$ and $g(x) = x^2 + x$:

35. Evaluate $g(x) - f(x)$

$$\begin{aligned} x^2 + x - (2 - x) \\ x^2 + x - 2 + x \\ x^2 + 2x - 2 \end{aligned}$$

36. Evaluate $f(x) \cdot g(x)$

$$\begin{aligned} (2 - x)(x^2 + x) \\ 2x^2 + 2x - x^3 - x^2 \\ -x^3 + x^2 + 2x \end{aligned}$$

37. Evaluate $g(f(5)) = g(2 - 5) = g(-3) = (-3)^2 + (-3) = 6$

38. Evaluate $f(g(x)) = f(x^2 + x) = 2 - (x^2 + x) = 2 - x^2 - x$

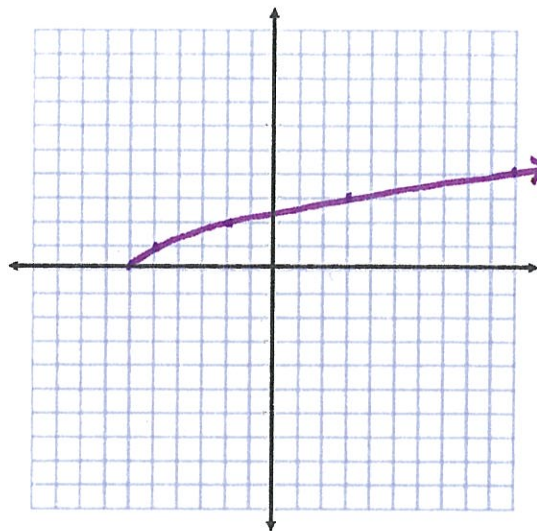
39. Evaluate $g(f(x)) = g(2 - x) = (2 - x)^2 + (2 - x)$
 $= (2 - x)(2 - x) + (2 - x)$
 $= 4 - 2x - 2x + x^2 + 2 - x$
 $= x^2 - 5x + 6$

Name:

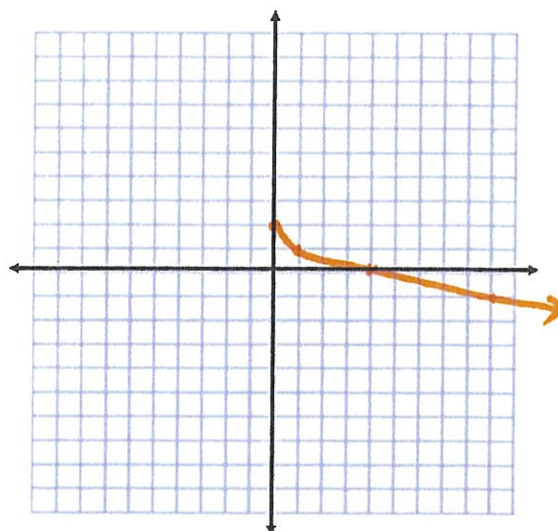
Solutions / Answers

For #'s 1-12, neatly and carefully graph each transformation of a parent function:

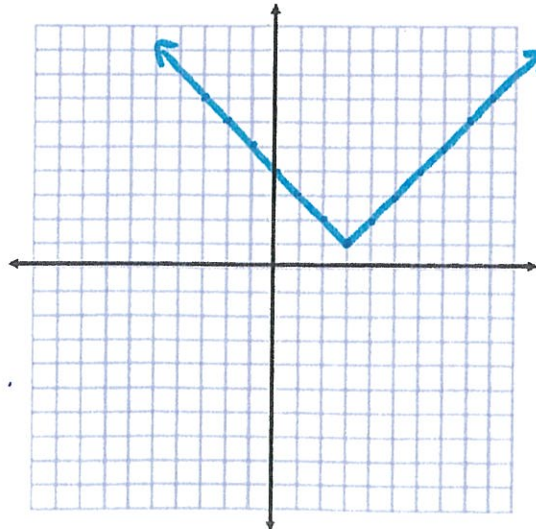
1. $f(x) = \sqrt{x+6}$ or $f(x) = (x+6)^{\frac{1}{2}}$



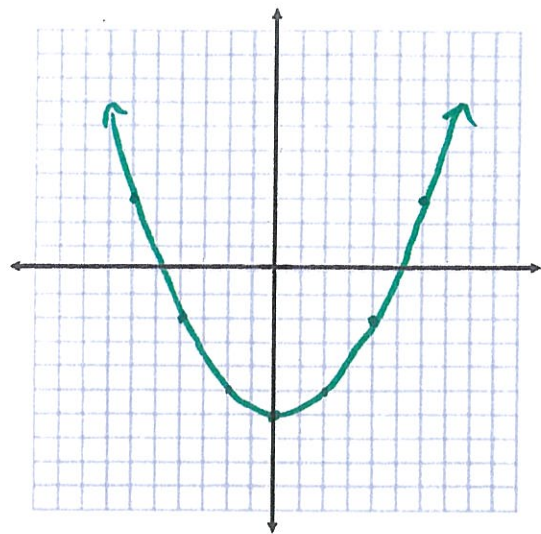
2. $f(x) = -\sqrt{x} + 2$ or $f(x) = -(x)^{\frac{1}{2}} + 2$



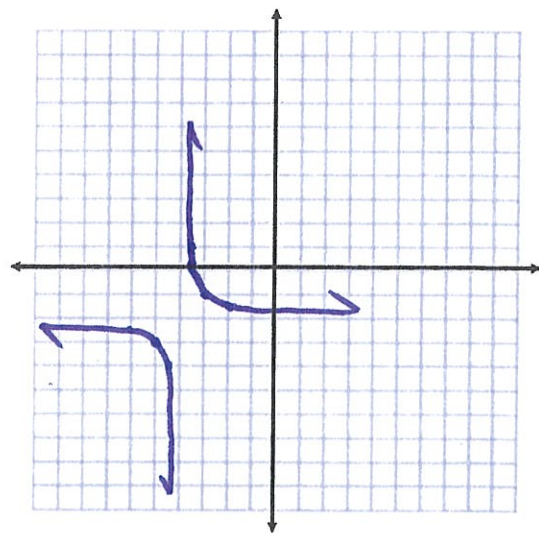
3. $f(x) = |x-3| + 1$



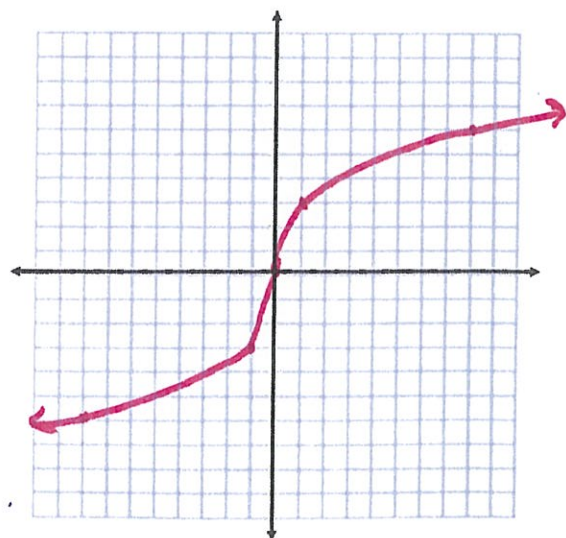
4. $f(x) = \left(\frac{1}{2}x\right)^2 - 6$



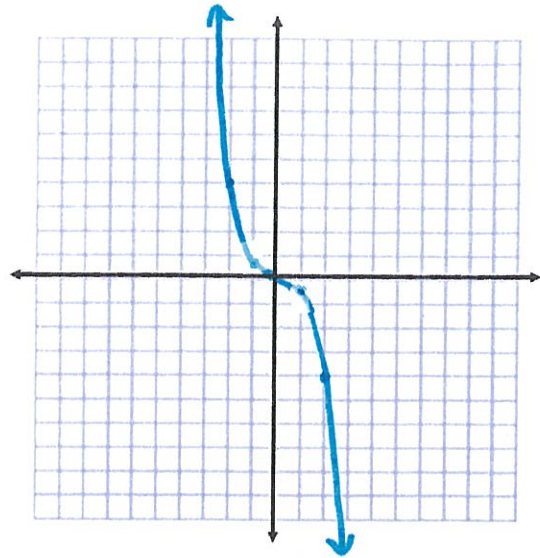
5. $f(x) = \frac{1}{x+4} - 2$



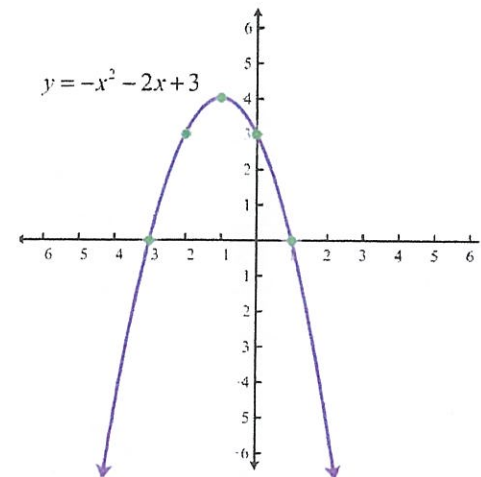
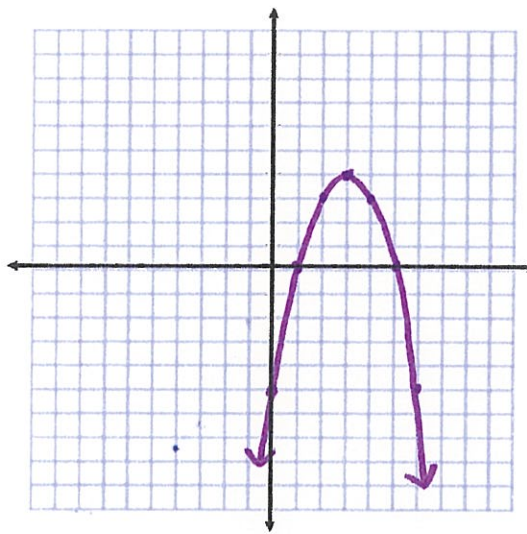
6. $f(x) = 3\sqrt[3]{x}$ or $f(x) = 3(x)^{\frac{1}{3}}$



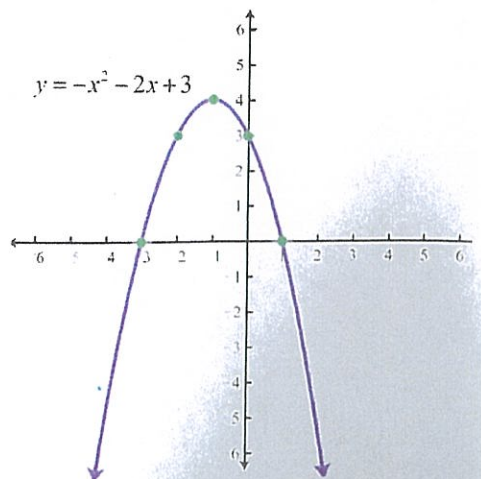
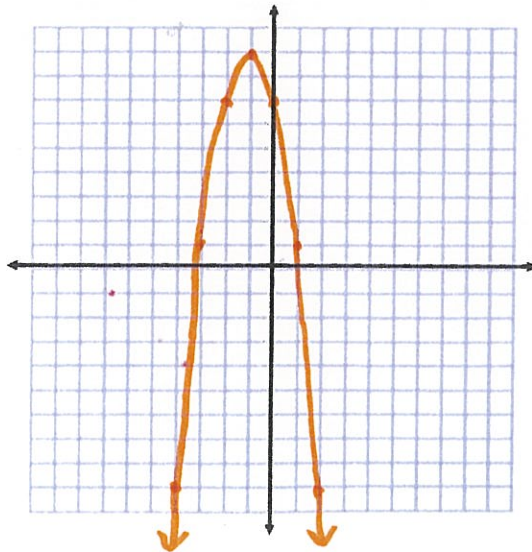
7. $f(x) = \frac{1}{2}(-x)^3$



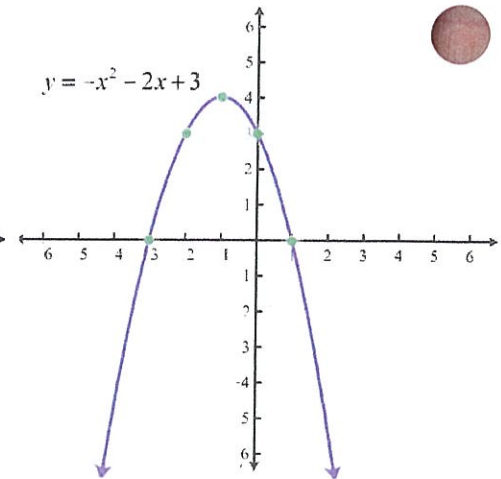
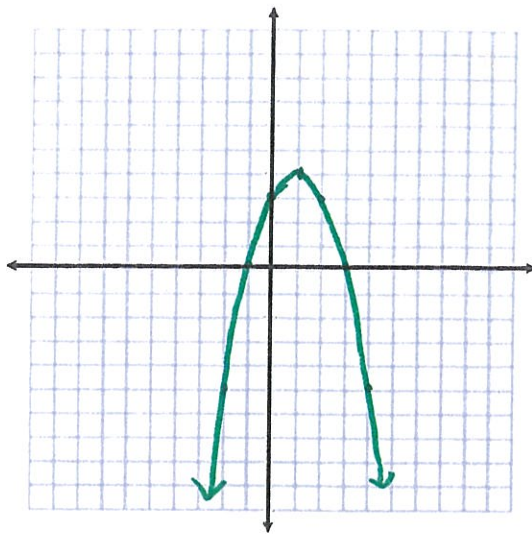
8. $T(x) = f(x-4)$



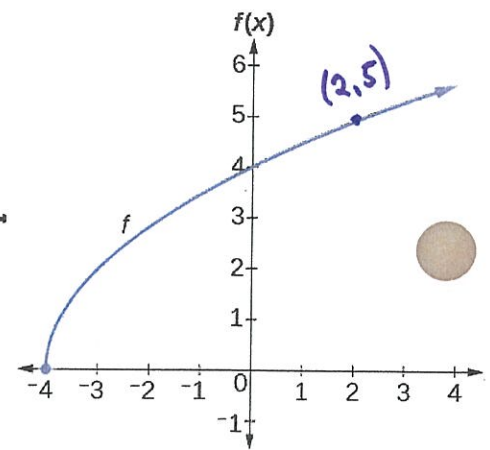
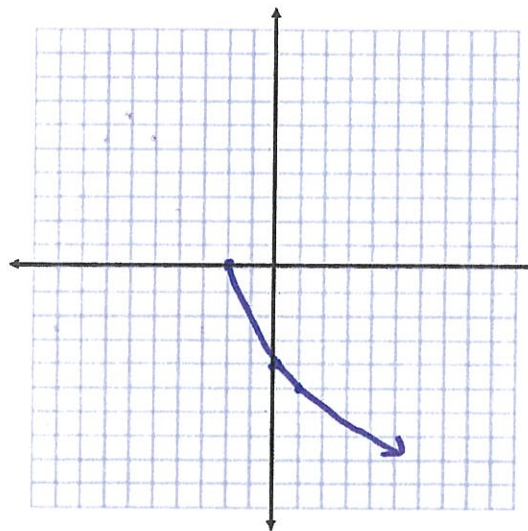
9. $T(x) = 2f(x) + 1$



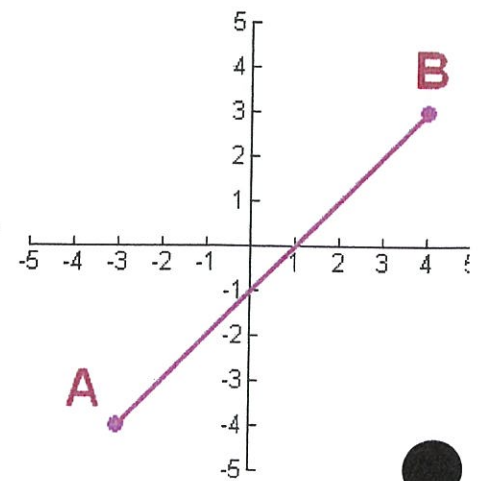
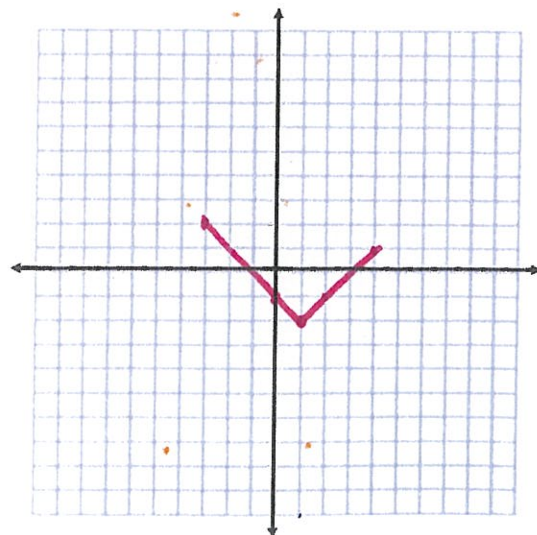
10. $T(x) = f(-x)$



11. $T(x) = -f(2x)$

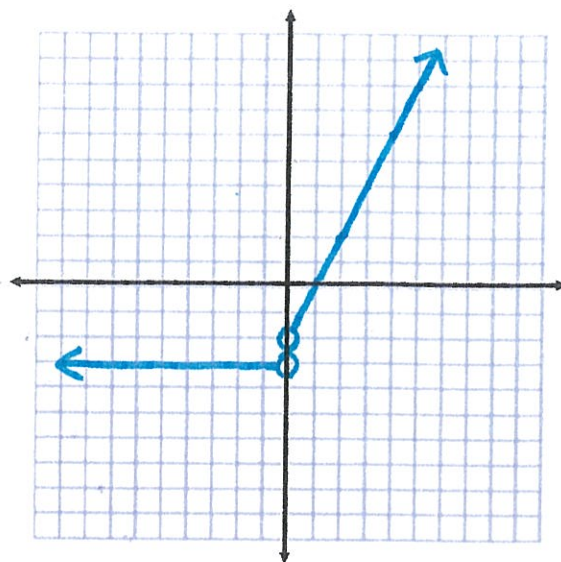


12. $T(x) = |f(x)| - 2$

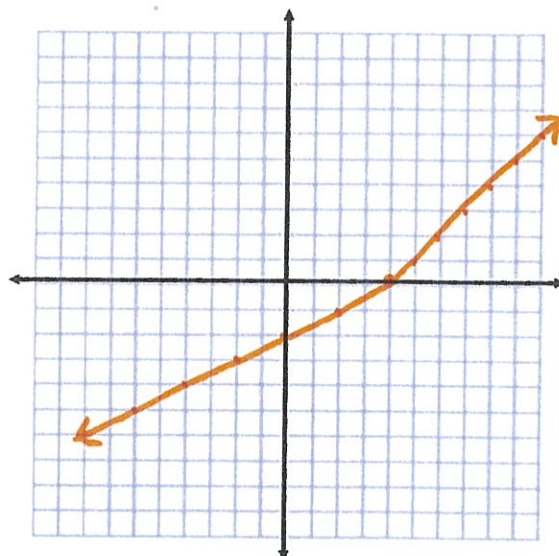


For #'s 13-15, neatly and carefully graph each piecewise function:

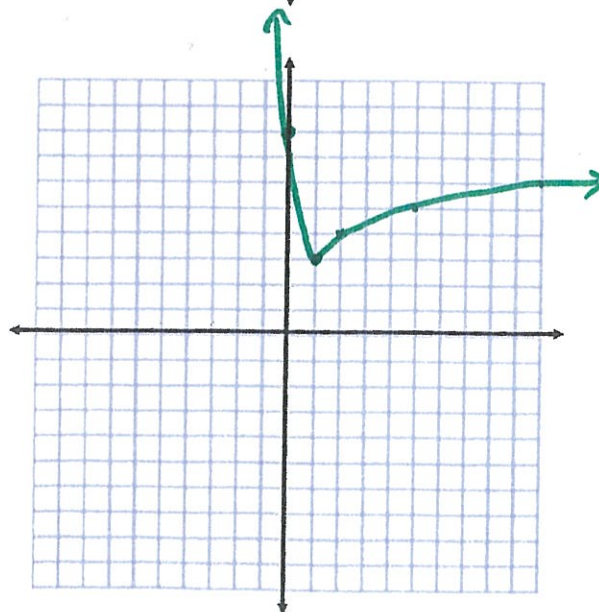
$$13. f(x) = \begin{cases} 2x-2 & \text{if } x > 0 \\ -3 & \text{if } x < 0 \end{cases}$$



$$14. f(x) = \begin{cases} |x-4| & \text{if } x > 4 \\ \frac{1}{2}x - 2 & \text{if } x \leq 4 \end{cases}$$

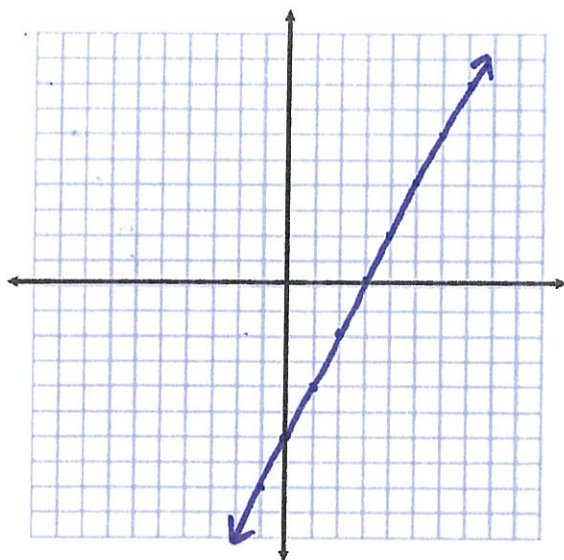


$$15. f(x) = \begin{cases} (x-3)^2 - 1 & \text{if } x < 1 \\ \sqrt{x-1} + 3 & \text{if } x \geq 1 \end{cases}$$

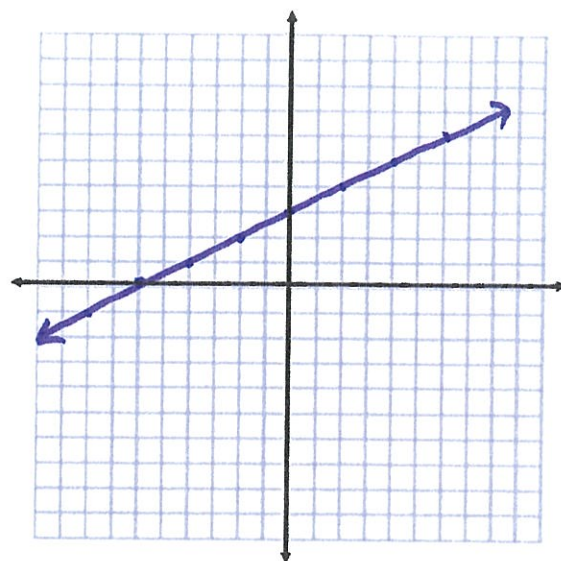


For #'s 16-18, neatly and carefully graph each original function and its inverse function:

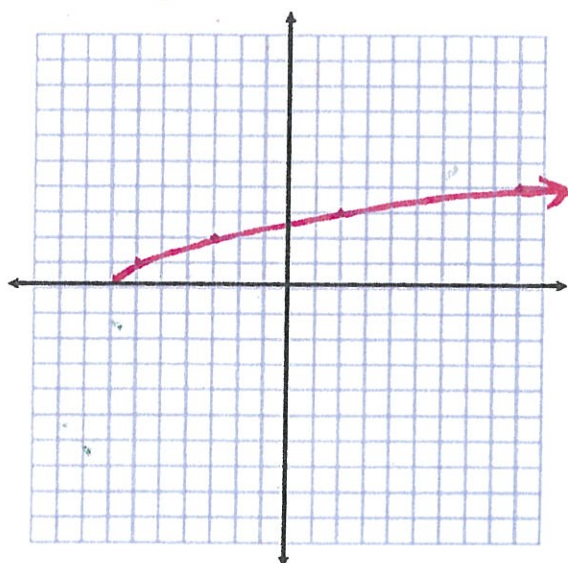
16. Graph $f(x) = 2x - 6$ on the grid below.



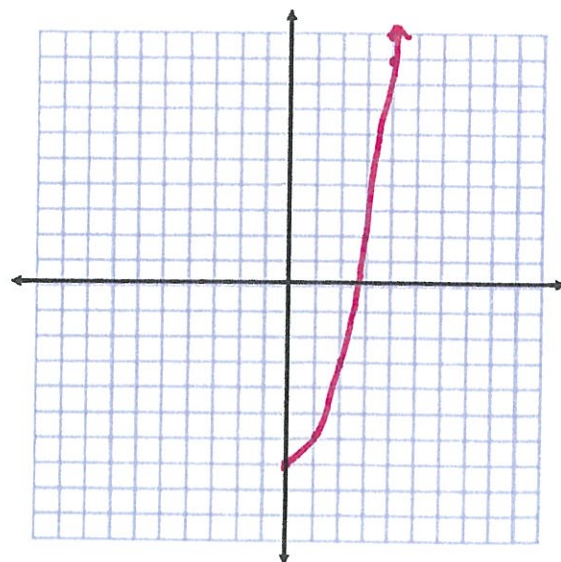
Graph the Inverse function below.



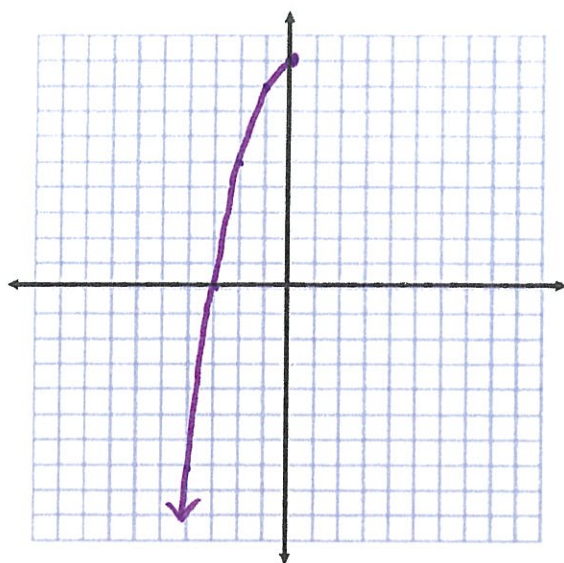
17. Graph $f(x) = \sqrt{x + 7}$ on the grid below.



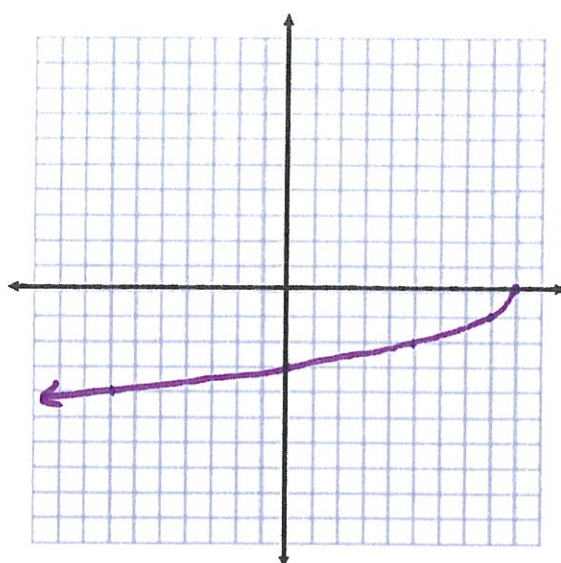
Graph the Inverse function below.



18. Graph $f(x) = 9 - x^2$ if $x \leq 0$ on the grid below.



Graph the Inverse function below.



For #'s 19-21, determine an equation for the inverse function algebraically (pay special attention to the range of the original function because it gives you the domain of the inverse function)

19. $f(x) = 2x - 6$

Range: $(-\infty, \infty)$

$f^{-1}(x) = \frac{1}{2}x + 3$

$y = 2x - 6$

$x = \frac{y + 6}{2}$

$x + 6 = 2y$

$\frac{x + 6}{2} = y$

$y = \frac{1}{2}x + 3$

Domain: $(-\infty, \infty)$

20. $f(x) = \frac{3}{2}x + 9$

Range: $(-\infty, \infty)$

$f^{-1}(x) = \frac{2}{3}x - 6$

$y = \frac{3}{2}x + 9$

$x = \frac{2}{3}(y - 9)$

$2x = 2y - 18$

$2x - 18 = 2y$ $y = \frac{2}{3}x - 6$

Domain: $(-\infty, \infty)$

21. $f(x) = \sqrt{x + 5}$

Range: $y \geq 0$

$f^{-1}(x) = x^2 - 5$

Domain: $x \geq 0$

$y = \sqrt{x + 5}$

$x = y^2 - 5$

$x^2 = y + 5$

$x^2 - 5 = y$

Name: Solutions / Answers

1. Factor $5y^2 + 12y + 7$

$$(5y + 7)(y + 1)$$

2. Factor $14t^2 + 11t - 15$

$$(2t + 3)(7t - 5)$$

3. Factor $x^2 - 25$

$$(x - 5)(x + 5)$$

4. Factor $n^2 + 8n + 16$

$$(n + 4)(n + 4) \\ (n + 4)^2$$

5. Factor $4w^2 - 20w + 25$

$$(2w - 5)(2w - 5) \\ (2w - 5)^2$$

6. Factor $m^3 - 64$

$$(m - 4)(m^2 + 4m + 16)$$

7. Factor $8k^3 + 27 = (2k)^3 + 3^3 = (2k + 3)(4k^2 - 6k + 9)$

8. Factor $8x^2 - 24x$ by factoring out the GCF initially

$$\frac{8x^2}{8x} - \frac{24x}{8x} \quad \downarrow 8x$$
$$8x(x-3)$$

9. Factor $5m^3 - 45m$ by factoring out the GCF initially

$$\frac{5m^3}{5m} - \frac{45m}{5m} \quad \downarrow 5m$$
$$5m(m^2-9)$$
$$5m(m-3)(m+3)$$

10. Factor $3y^3 + 6y^2 - 45y$ by factoring out the GCF initially

$$\frac{3y^3}{3y} + \frac{6y^2}{3y} - \frac{45y}{3y} \quad \downarrow 3y$$
$$3y(y^2 + 2y - 15)$$
$$3y(y-3)(y+5)$$

11. Factor $4n^4 - 20n^3 - 56n^2$ by factoring out the GCF initially

$$\frac{4n^4}{4n^2} - \frac{20n^3}{4n^2} - \frac{56n^2}{4n^2} \quad \downarrow 4n^2$$
$$4n^2(n^2 - 5n - 14)$$
$$4n^2(n-7)(n+2)$$

12. Factor $x^3 + 5x^2 + 4x + 20$ by grouping

$$x^2(x+5) + 4(x+5)$$
$$(x+5)(x^2+4)$$

13. Factor $y^3 + 2y^2 - 9y - 18$ by grouping initially

$$y^2(y+2) - 9(y+2)$$
$$(y+2)(y^2-9)$$
$$(y+2)(y-3)(y+3)$$

14. Factor $z^4 - 13z^2 + 36 = (z^2 - 4)(z^2 - 9)$
 $= (z-2)(z+2)(z-3)(z+3)$

15. Solve by factoring $3x^2 - 11x - 4 = 0$

$$(3x + 1)(x - 4) = 0$$

$$\begin{array}{l} 3x + 1 = 0 \\ 3x = -1 \\ x = -\frac{1}{3} \end{array} \quad \begin{array}{l} x - 4 = 0 \\ x = 4 \end{array}$$

Solution set
 $\{-\frac{1}{3}, 4\}$

16. Solve by factoring $x^2 - 81 = 0$

$$(x - 9)(x + 9) = 0$$

Solution set
 $\{9, -9\}$

17. Solve by factoring $9k^2 - 49 = 0$

$$(3k - 7)(3k + 7) = 0$$

$$\begin{array}{l} 3k - 7 = 0 \\ k = \frac{7}{3} \end{array} \quad \begin{array}{l} 3k + 7 = 0 \\ k = -\frac{7}{3} \end{array}$$

Solution set
 $\{\frac{7}{3}, -\frac{7}{3}\}$

18. Solve by factoring $p^2 - 20p + 100 = 0$

$$(p - 10)(p - 10) = 0$$

$$(p - 10)^2 = 0$$

$$p - 10 = 0 \quad p = 10$$

Solution set
 $\{10\}$

19. Solve by factoring $m^3 - 27 = 0$

$$(m - 3)(m^2 + 3m + 9) = 0$$

$$\begin{array}{l} \downarrow \\ m - 3 = 0 \\ m = 3 \end{array}$$

$$m = \frac{-3 \pm \sqrt{9 - 4(1)(9)}}{2(1)}$$

$$m = \frac{-3 \pm \sqrt{9 - 36}}{2}$$

$$\begin{array}{l} m = \frac{-3 \pm \sqrt{-27}}{2} \\ m = \frac{-3 \pm 3i\sqrt{3}}{2} \end{array}$$

$$\left\{ 3, \frac{-3 + 3i\sqrt{3}}{2}, \frac{-3 - 3i\sqrt{3}}{2} \right\}$$

20. Solve by factoring $2m^3 - 32m = 0$

$$\begin{aligned} 2m(m^2 - 16) &= 0 \\ 2m(m - 4)(m + 4) &= 0 \\ \downarrow \quad \quad \downarrow \quad \quad \downarrow \\ 2m = 0 \quad m - 4 = 0 \quad m + 4 = 0 \\ m = 0 \quad m = 4 \quad m = -4 \end{aligned}$$

Solution Set
 $\{0, 4, -4\}$

21. Solve by factoring $2x^2 + 10x + 12 = 0$ by first factoring out the GCF

$$\begin{aligned} 2(x^2 + 5x + 6) &= 0 \\ 2(x + 2)(x + 3) &= 0 \\ x + 2 = 0 \quad x + 3 = 0 \\ x = -2 \quad x = -3 \end{aligned}$$

Solution Set
 $\{-2, -3\}$

22. Solve by factoring $2y^3 - 6y^2 - 36y = 0$ by first factoring out the GCF

$$\begin{aligned} 2y(y^2 - 3y - 18) &= 0 \\ 2y(y - 6)(y + 3) &= 0 \\ y = 0 \quad y = 6 \quad y = -3 \end{aligned}$$

Solution Set
 $\{0, 6, -3\}$

23. Solve by factoring (by grouping) $y^3 + 2y^2 - 25y - 50 = 0$

$$\begin{aligned} y^2(y + 2) - 25(y + 2) &= 0 \\ (y + 2)(y^2 - 25) &= 0 \\ (y + 2)(y - 5)(y + 5) &= 0 \\ y = -2 \quad y = 5 \quad y = -5 \end{aligned}$$

Solution Set
 $\{-2, 5, -5\}$

24. Solve by factoring $z^4 - 20z^2 + 64 = 0$

$$\begin{aligned} (z^2 - 4)(z^2 - 16) &= 0 \\ (z - 2)(z + 2)(z - 4)(z + 4) &= 0 \\ z = 2 \quad z = -2 \quad z = 4 \quad z = -4 \end{aligned}$$

Solution Set
 $\{2, -2, 4, -4\}$

25. A polynomial function of degree 2 has the solution set $\{-8, 5\}$. Determine an equation of the function in **standard form**.

$$\begin{aligned} (x + 8)(x - 5) &= 0 \\ x^2 + 3x - 40 &= 0 \end{aligned}$$

26. A polynomial function of degree 3 has the solution set $\{2, -3\}$ with -3 as a double root. Determine an equation of the function in **standard form**.

$$\begin{aligned}(x-2)(x+3)(x+3) &= 0 \\ (x-2)(x^2+6x+9) &= 0 \\ x^3+4x^2-3x-18 &= 0\end{aligned}$$

27. A polynomial function of degree 3 has the solution set $\left\{0, -5, \frac{2}{3}\right\}$. Determine an equation of the function in **standard form**.

$$\begin{aligned}x=0 \quad x=-5 \quad x=\frac{2}{3} \\ x=0 \quad x+5=0 \quad 3x-2=0 \\ x(x+5)(3x-2) &= 0 \\ x(3x^2+13x-10) &= 0 \\ 3x^3+13x^2-10x &= 0\end{aligned}$$

28. Solve $x^2+16=0$

$$\begin{aligned}x^2 &= -16 \\ \sqrt{x^2} &= \pm \sqrt{-16} \\ x &= \pm i\sqrt{16} \\ x &= \pm i \cdot 4\end{aligned}$$

solution set
 $\{4i, -4i\}$

29. Solve $x^2-10=0$

$$\begin{aligned}x^2 &= 10 \\ \sqrt{x^2} &= \pm \sqrt{10} \\ x &= \pm \sqrt{10}\end{aligned}$$

solution set
 $\{\sqrt{10}, -\sqrt{10}\}$

30. Solve $k^2-12=0$

$$\begin{aligned}k^2 &= 12 \\ \sqrt{k^2} &= \pm \sqrt{12} \\ k &= \pm \sqrt{4 \cdot 3} \\ k &= \pm 2\sqrt{3}\end{aligned}$$

solution set
 $\{2\sqrt{3}, -2\sqrt{3}\}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

31. Solve $w^2 - 4w + 5 = 0$ using the quadratic formula

$$w = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$w = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$w = \frac{4 \pm \sqrt{-4}}{2}$$

$$w = \frac{4 \pm 2i}{2}$$

$$w = 2 \pm i$$

solution set
 $\{2+i, 2-i\}$

32. Solve $k^3 + 64 = 0$ by factoring the sum of cubes and by using the quadratic formula

$$(k+4)(k^2 - 4k + 16) = 0$$

$$k+4=0$$

$$k = -4$$

$$k = \frac{4 \pm \sqrt{16 - 64}}{2}$$

$$k = \frac{4 \pm \sqrt{-48}}{2}$$

$$k = \frac{4 \pm 4i\sqrt{3}}{2}$$

solution set

$\{-4, 2+2i\sqrt{3}, 2-2i\sqrt{3}\}$

33. Solve $5m^3 + 45m = 0$ by factoring out the GCF and using other methods

$$5m(m^2 + 9) = 0$$

$$5m = 0 \quad m^2 + 9 = 0$$

$$m = 0$$

$$m^2 = -9$$

$$\sqrt{m^2} = \pm \sqrt{-9}$$

$$m = \pm 3i$$

solution set

$\{0, 3i, -3i\}$

34. Solve $2x^2 + 12x + 14 = 0$ by factoring out the GCF and using the quadratic formula

$$2(x^2 + 6x + 7) = 0$$

$$a=1 \quad b=6 \quad c=7$$

$$x = \frac{-6 \pm \sqrt{36 - 28}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{2}}{2}$$

$$x = \frac{-6 \pm \sqrt{8}}{2}$$

solution set
 $\{-3+\sqrt{2}, -3-\sqrt{2}\}$

35. Solve $x^3 + 3x^2 + 4x + 12 = 0$ by grouping initially

$$\sqrt{-1} = i$$

Mr. Michael T. Davis
WLPCS Pre-Calculus

Units 2.5-2.7 Review for Final Exam
May 21, 2018

Name: Solutions / Answers

1. Solve $x^2 + 16 = 0$

$$\begin{aligned} x^2 &= -16 \\ \sqrt{x^2} &= \pm \sqrt{-16} \\ x &= \pm i\sqrt{16} \quad x = \pm i4 \end{aligned}$$

$$\{4i, -4i\}$$

2. Solve $k^3 - 5k = 0$

$$\begin{aligned} k(k^2 - 5) &= 0 \\ \downarrow \quad \downarrow & \\ k=0 \quad k^2 - 5 = 0 & \quad \sqrt{k^2} = \pm \sqrt{5} \\ k^2 = 5 \quad k &= \pm \sqrt{5} \end{aligned}$$

$$\{0, \sqrt{5}, -\sqrt{5}\}$$

3. Solve $p^2 + 6p + 10 = 0$.

Quadratic Formula

$$p = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$p = \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$p = \frac{-6 \pm \sqrt{-4}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$p = \frac{-6 \pm 2i}{2}$$

$$p = -3 \pm i$$

$$\{-3 + i, -3 - i\}$$

4. Solve $x^3 - 2x^2 - 12x + 24 = 0$

$$x^2(x-2) - 12(x-2) = 0$$

$$(x-2)(x^2 - 12) = 0$$

$$x-2=0 \quad x^2 - 12 = 0$$

$$x = 2$$

$$x^2 = 12$$

$$\sqrt{x^2} = \pm \sqrt{12} \quad x = \pm 2\sqrt{3}$$

$$\{2, 2\sqrt{3}, -2\sqrt{3}\}$$

5. Write an equation (in standard form) of the polynomial function with solution set $\{10i, -10i\}$ and with degree 2.

$$(x - 10i)(x - (-10i)) = 0$$

$$(x - 10i)(x + 10i) = 0$$

$$x^2 + 10i - 10i - 100i^2 = 0$$

$$x^2 - 100(-1) = 0$$

$$x^2 + 100 = 0$$

$$i^2 = -1$$

6. Write an equation (in standard form) of the polynomial function with solution set $\{-2, \sqrt{7}, -\sqrt{7}\}$ and with degree 3.

$$\begin{aligned}(x+2)(x-\sqrt{7})(x+\sqrt{7}) &= 0 \\(x+2)(x^2 + \cancel{\sqrt{7}x} - \cancel{\sqrt{7}x} - \sqrt{49}) &= 0 \\(x+2)(x^2 - 7) &= 0 \\x^3 + 2x^2 - 7x - 14 &= 0\end{aligned}$$

7. Simplify $\frac{8x+24}{4} = 2x+6$

8. Simplify $\frac{2x^2-5x-12}{2x^2+13x+15} = \frac{\cancel{(2x+3)}(x-4)}{\cancel{(2x+3)}(x+5)} = \frac{x-4}{x+5}$

9. Simplify $\frac{m^2-49}{m^2-14m+49} \cdot \frac{m^2-13m+42}{m^2+7m} = \frac{\cancel{(m-7)}\cancel{(m+7)}}{\cancel{(m-7)}\cancel{(m-7)}} \cdot \frac{(m-6)\cancel{(m-7)}}{m\cancel{(m+7)}}$
 $= \frac{m-6}{m}$

10. Simplify $\frac{p^3+1}{p^3-8} \div \frac{p+1}{p-2} = \frac{\cancel{(p+1)}(p^2-p+1)}{\cancel{(p-2)}(p^2+2p+4)} \cdot \frac{\cancel{p-2}}{\cancel{p+1}} = \frac{p^2-p+1}{p^2+2p+4}$

11. Solve $\frac{5x-70}{x-14} = 5$

$$5x - 70 = 5x - 70$$

$$0 = 0$$

All real numbers, $x \neq 14$

12. Solve $\frac{6x-78}{x-13} = 7$

$$6x - 78 = 7x - 91$$

$$13 \neq x$$

No solution

13. Solve $\frac{2}{3} = \frac{5w+4}{6+7w}$

$$\frac{2}{3} (6+7w)(3) = \frac{5w+4}{6+7w} (6+7w)(3)$$

$$12 + 14w = 15w + 12$$

$$0 = w$$

$\{0\}$ solution set

14. Solve $\frac{21}{h^2} - 1 = \frac{4}{h}$

$$h^2 \left(\frac{21}{h^2} \right) - h^2(1) = h^2 \left(\frac{4}{h} \right)$$

$$21 - h^2 = 4h$$

$$0 = h^2 + 4h - 21$$

$$0 = (h+7)(h-3)$$

$$h = -7 \quad h = 3$$

$\{-7, 3\}$ solution set

15. Solve $x = \frac{33}{x} - 8$

$$x(x) = x\left(\frac{33}{x}\right) - x(8)$$

$$x^2 = 33 - 8x$$

$$x^2 + 8x - 33 = 0$$

$$(x+11)(x-3) = 0$$

$$x = -11 \quad x = 3$$

$\{-11, 3\}$ solution set

16. Solve $1 + \frac{2}{y} = \frac{4}{y-3}$

$$y(y-3)(1) + \cancel{y(y-3)}\left(\frac{2}{y}\right) = y(y-3)\left(\frac{4}{y-3}\right)$$

$$y^2 - 3y + 2y - 6 = 4y$$

$$y^2 - 5y - 6 = 0$$

$$(y-6)(y+1) = 0$$

$$y = 6 \quad y = -1$$

$\{6, -1\}$ solution set

17. Solve $\frac{2}{x+1} - \frac{1}{x} = \frac{1}{6}$

$$6x(\cancel{x+1})\left(\frac{2}{\cancel{x+1}}\right) - 6\cancel{x}(x+1)\left(\frac{1}{\cancel{x}}\right) = 6x(x+1)\left(\frac{1}{6}\right)$$

$$12x - 6x - 6 = x^2 + x$$

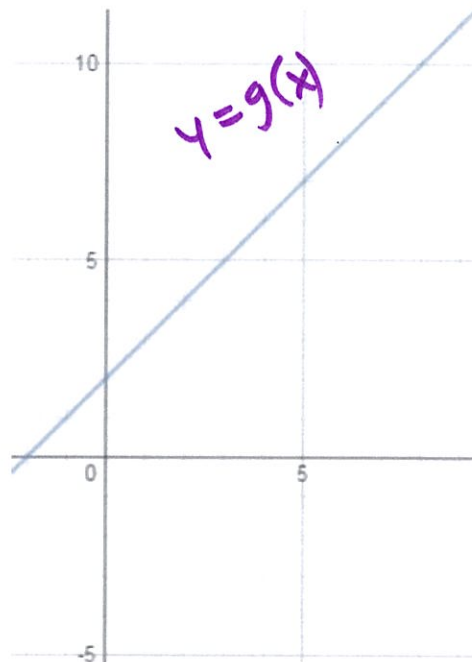
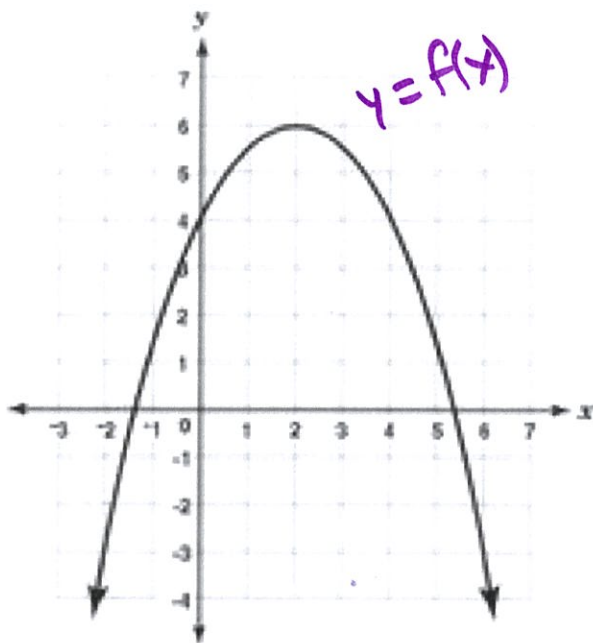
$$0 = x^2 - 5x + 6$$

$$0 = (x-2)(x-3)$$

$$x = 2 \quad x = 3$$

$\{2, 3\}$ solution set

Use the graphs of $y = f(x)$ on the left and $y = g(x)$ on the right to answer questions # 40-43



40. Evaluate $g(f(2)) = g(6) = 8$

41. Evaluate $f(0) + g(0) = 4$

42. Evaluate $f(-2) \cdot g(-2) = (-3)(0) = 0$

43. Evaluate $\frac{g(2)}{f(2)} = \frac{5}{6}$

Name: Answers / Solutions

Directions for # 1-4: For each rational function:

1. Identify an equation for each vertical asymptote (VA), if any exist.
2. Identify an equation for each horizontal asymptote (HA), if any exist.
3. Identify the coordinates of all x-intercepts, if any exist.
4. Identify the coordinates of a y-intercept, if one exists.
5. Identify any value of x for which the graph has a hole.
6. Draw a neat and accurate graph of the function.

1. (8 pts) $f(x) = \frac{8}{x-2}$

VA: $x = 2$

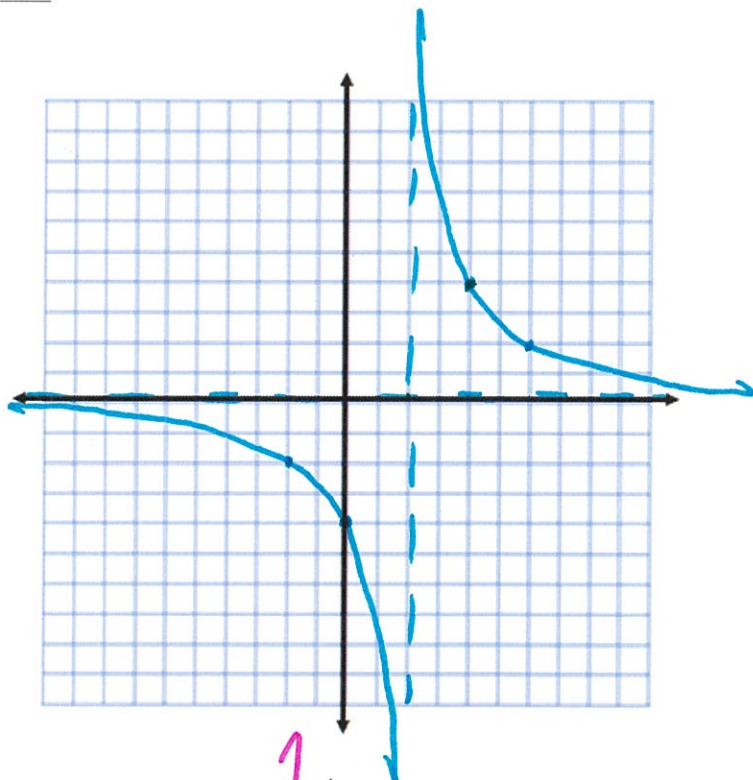
HA: $y = 0$ x-axis

X-Intercept(s): None

Y-Intercept: $(0, -4)$

X-Value of Hole: None

$$\begin{array}{c|c} x & y \\ \hline 6 & 2 \end{array}$$



2. (8 pts) $f(x) = \frac{x-4}{x+1}$

VA: $x = -1$

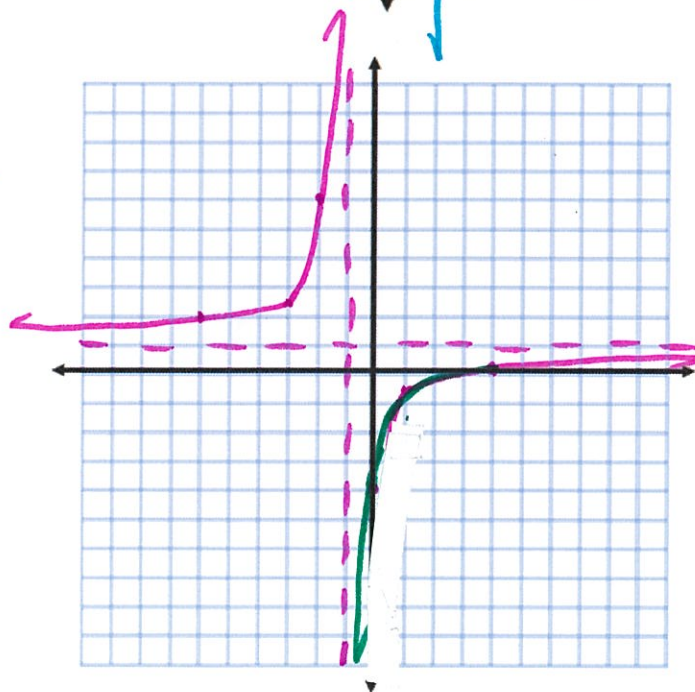
HA: $y = 1$

X-Intercept(s): $(4, 0)$

Y-Intercept: $(0, -4)$

X-Value of Hole: None

$$\begin{array}{c|c} x & y \\ \hline 1 & -\frac{3}{2} \end{array}$$



3. (8 pts) $f(x) = \frac{x^2 - 5x + 6}{x - 3}$ or $f(x) = \frac{(x-3)(x-2)}{x-3}$

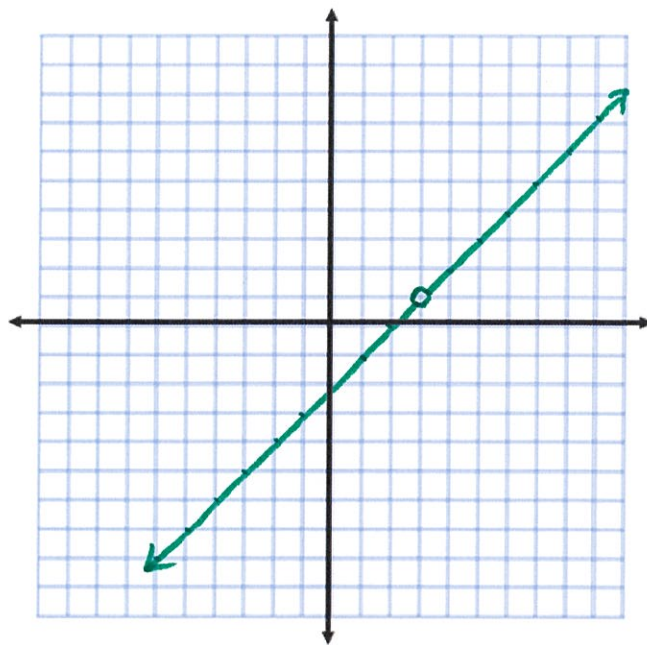
VA: **None**

HA: **None**

X-Intercept(s): **(2, 0)**

Y-Intercept: **(0, -2)**

X-Value of Hole: **x = 3**



4. (8 pts) $f(x) = \frac{x+2}{x^2+x-2}$ or $f(x) = \frac{x+2}{(x+2)(x-1)}$

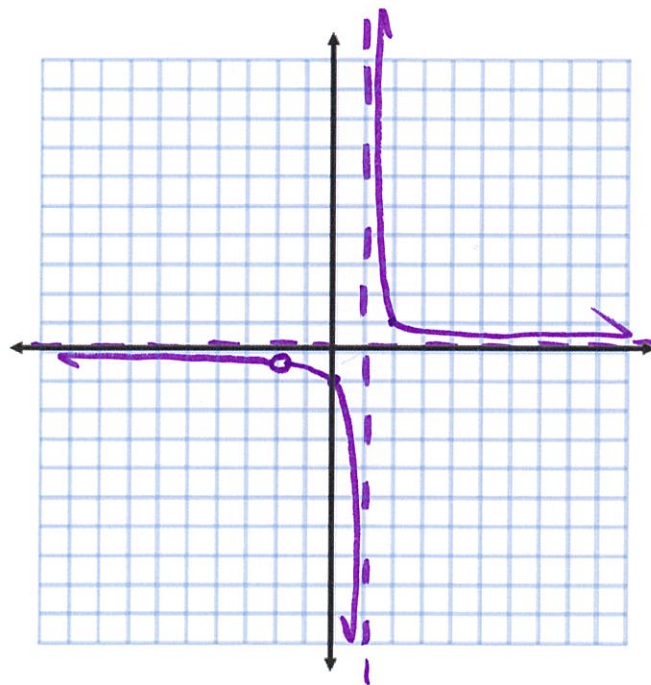
VA: **x = 1**

HA: **y = 0 x-axis**

X-Intercept(s): **None**

Y-Intercept: **(0, -1)**

X-Value of Hole: **x = -2**



Directions for #'s 5-11: Multiple Choice

5. (4 pts) The rational function $f(x) = \frac{x^2 - 9}{x^2 - 4x - 21}$ has the following domain:
- a. $D: \mathbb{R}, x \neq -3, x \neq 3$
 - b. $D: \mathbb{R}, x \neq -7, x \neq 3$
 - ☒ c. $D: \mathbb{R}, x \neq -3, x \neq 7$
 - d. $D: \mathbb{R}, x \neq -21, x \neq -4$
 - e. $D: \mathbb{R}$
6. (4 pts) The rational function $f(x) = \frac{x - 8}{x^2 - 25}$ has an x-intercept with coordinates:
- a. $(0, 8)$
 - b. $(5, 0)$
 - ☒ c. $(8, 0)$
 - d. $\left(0, \frac{8}{25}\right)$
 - e. There is no x-intercept
7. (4 pts) The rational function $f(x) = \frac{10}{x^2 - 4}$ has a horizontal asymptote with equation:
- ☒ a. $y = 0$
 - b. $x = 2$
 - c. $y = 10$
 - d. $x = -2$ & $x = 2$
 - e. There is no horizontal asymptote
8. (4 pts) The rational function $f(x) = \frac{x^2 - 8x + 16}{x^2 - 16}$ has a hole at:
- a. $y = -1$
 - b. $y = 1$
 - c. $x = -4$
 - ☒ d. $x = 4$
 - e. There is no hole
- $$f(x) = \frac{(x-4)(x-4)}{(x-4)(x+4)}$$

9. (4 pts) The rational function $f(x) = \frac{x-8}{x^2-16}$ has a y-intercept with coordinates:

- a. $(8,0)$
- b. $(0,2)$
- c. $(0,8)$
- ☒ d. $(0, \frac{1}{2})$
- e. There is no y-intercept

10. (4 pts) The rational function $f(x) = \frac{x^2-9x+14}{x^2+4x-12}$ has an x-intercept(s) with coordinates:

- a. $(2,0)$ & $(-6,0)$
- ☒ b. $(7,0)$ only
- c. $(2,0)$ only
- d. $(2,0)$ & $(7,0)$
- e. There is no x-intercept

$$f(x) = \frac{(x-7)(x-2)}{(x+6)(x-2)}$$

11. (4 pts) The rational function $f(x) = \frac{6x^2+3x-1}{2x^2-18}$ has a horizontal asymptote with equation:

- a. $y=6$
- b. $x=3$
- c. $y=0$
- ☒ d. $y=3$
- e. None of the above

Directions for # 12-16: For each rational function:

1. Identify an equation for each vertical asymptote (VA), if any exist.
2. Identify an equation for each horizontal asymptote (HA), if any exist.
3. Identify the coordinates of all x-intercepts, if any exist.
4. Identify the coordinates of a y-intercept, if one exists.
5. Identify any value of x for which the graph has a hole.
6. Write an equation for the rational function.

12. (8 pts)

VA: $x = -4$

HA: $y = 0$

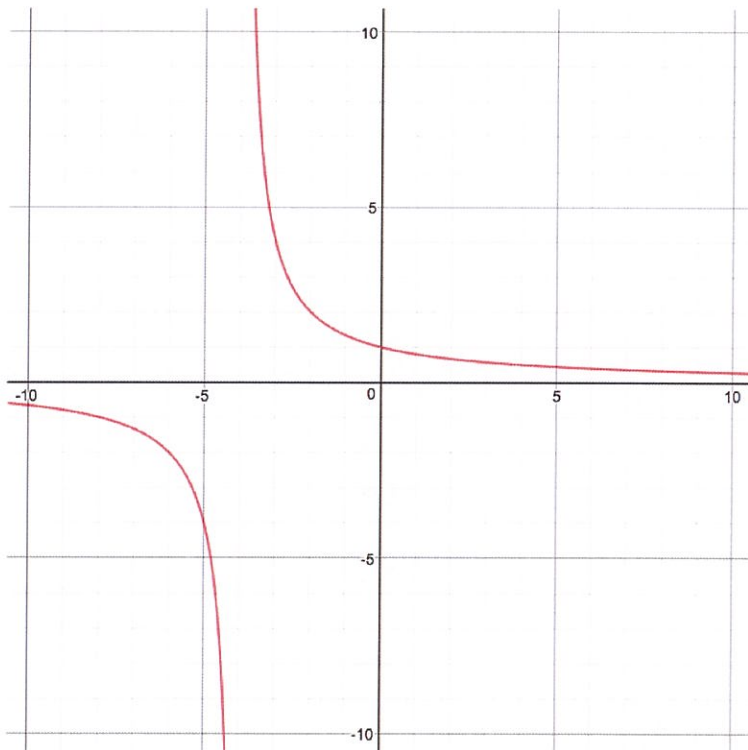
X-Intercept(s): *None*

Y-Intercept: $(0, 1)$

X-Value of Hole: *None*

Equation:

$$f(x) = \frac{4}{x+4}$$



13. (8 pts)

VA: $x = -1$

HA: $y = 3$

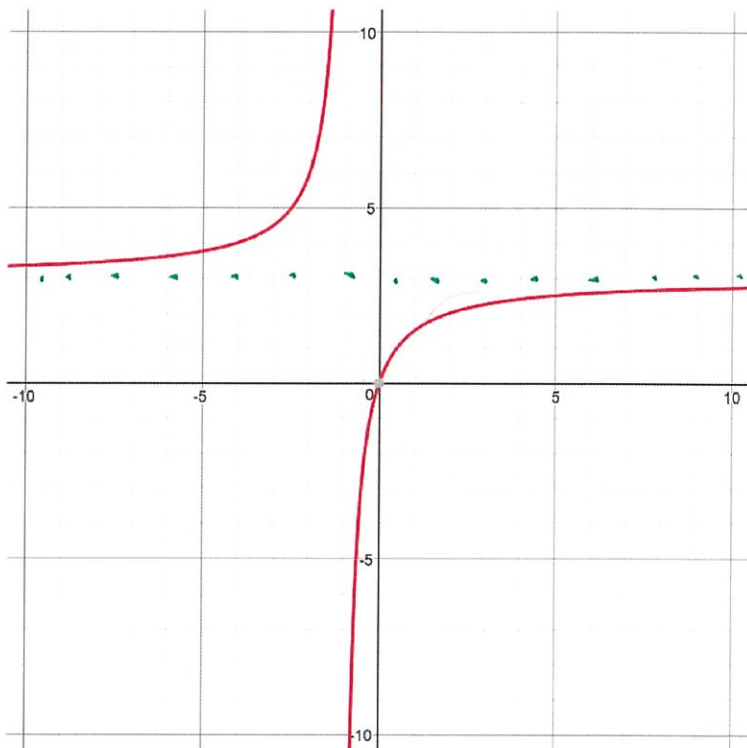
X-Intercept(s): $(0, 0)$

Y-Intercept: $(0, 0)$

X-Value of Hole: *None*

Equation:

$$g(x) = \frac{3x}{x+1}$$



14. (8 pts)

VA: $x = -2$

HA: $y = 2$

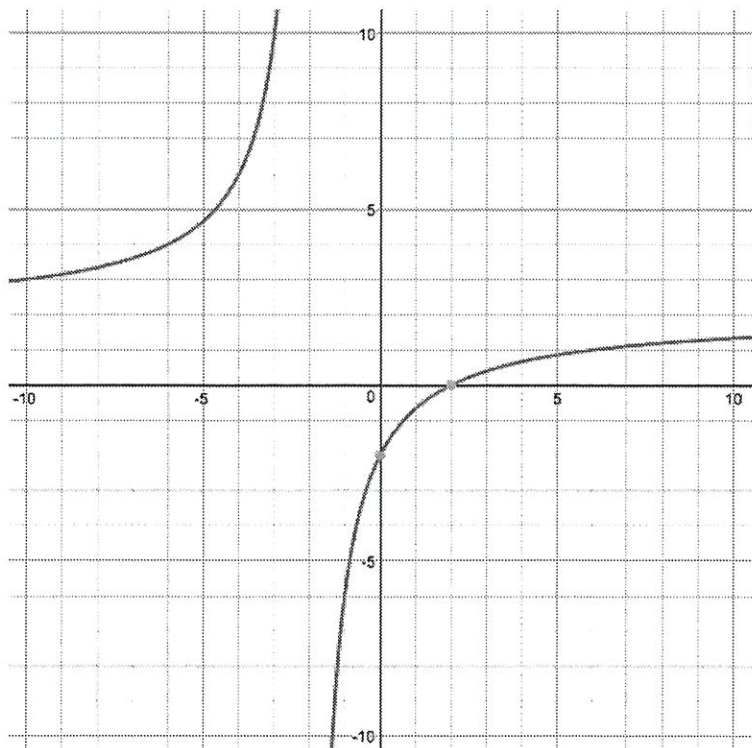
X-Intercept(s): $(2, 0)$

Y-Intercept: $(0, -2)$

X-Value of Hole: None

Equation:

$$f(x) = \frac{2(x-2)}{x+2}$$



15. (8 pts)

VA: None

HA: None

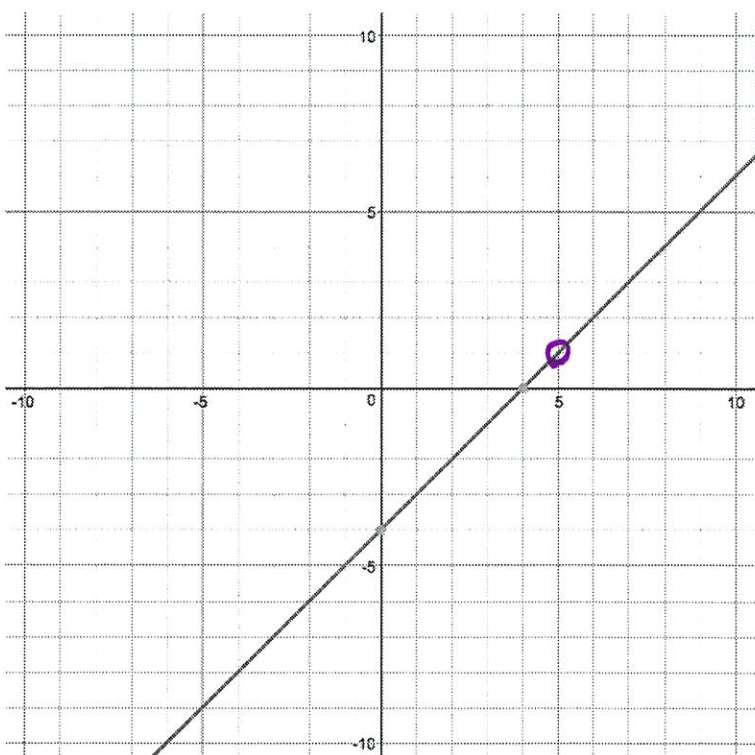
X-Intercept(s): $(4, 0)$

Y-Intercept: $(0, -4)$

X-Value of Hole: $x = 5$

Equation:

$$f(x) = \frac{(x-5)(x-4)}{(x-5)}$$



16. (8 pts)

VA: $x = -1$

HA: $y = 0$

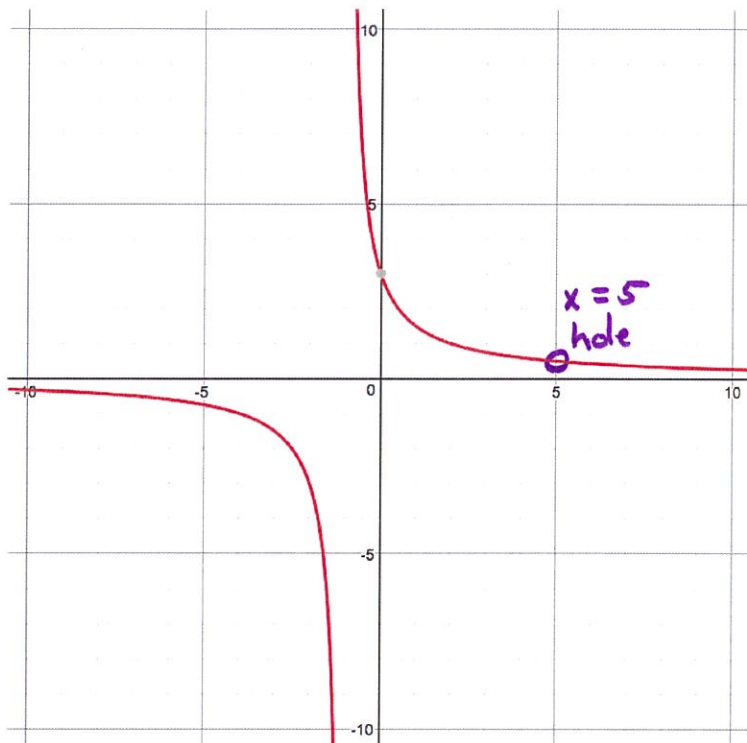
X-Intercept(s): *None*

Y-Intercept: $(0, 3)$

X-Value of Hole: $x = 5$

Equation:

$$K(x) = \frac{3(x-5)}{(x-5)(x+1)}$$



Optional Extra Credit

A. (8 pts) $f(x) = \frac{3x^2 - 3}{x^2 + 2x - 3} = \frac{3(x-1)(x+1)}{(x+3)(x-1)}$

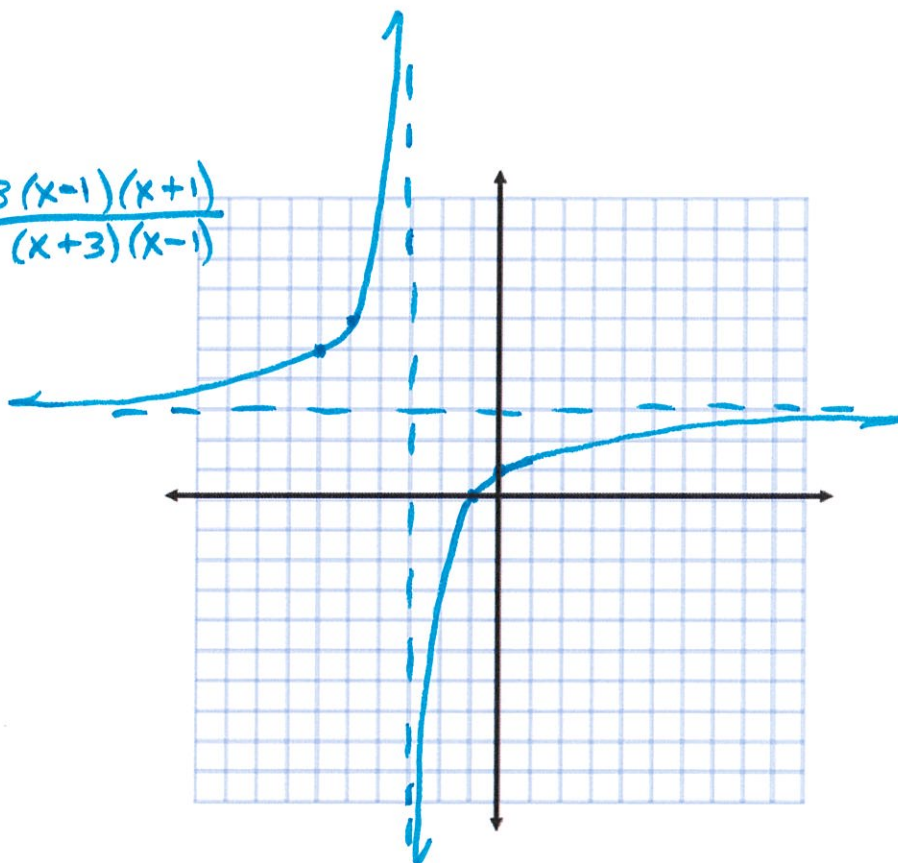
VA: $x = -3$

HA: $y = 3$

X-Intercept(s): $(-1, 0)$

Y-Intercept: $(0, 1)$

X-Value of Hole: $x = 1$



B. (8 pts)

VA: $x = 2$ and $x = -3$

HA: $y = 0$

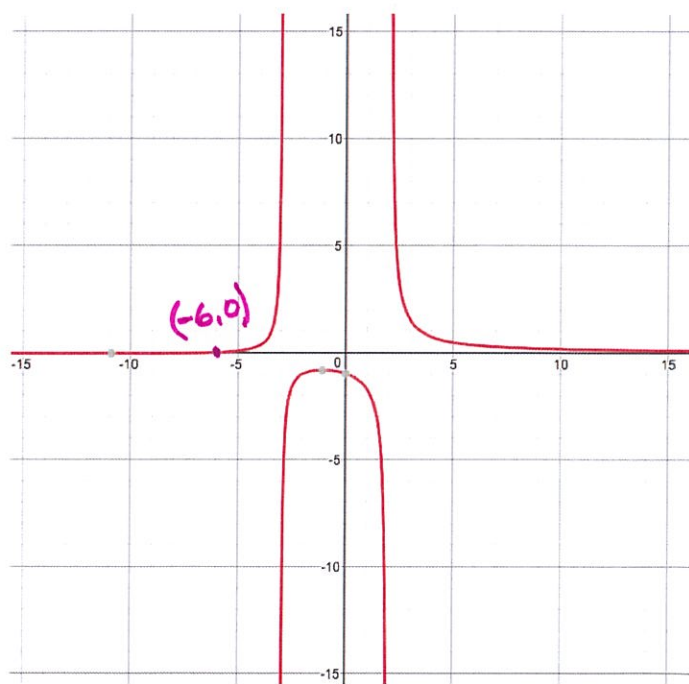
X-Intercept(s): $(-6, 0)$

Y-Intercept: $(0, -1)$

X-Value of Hole: *None*

Equation:

$$f(x) = \frac{x+6}{(x-2)(x+3)}$$

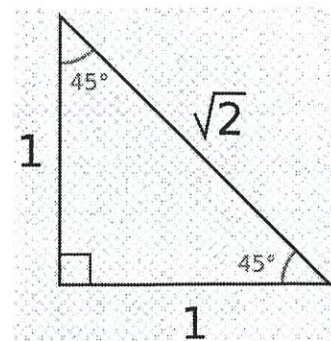
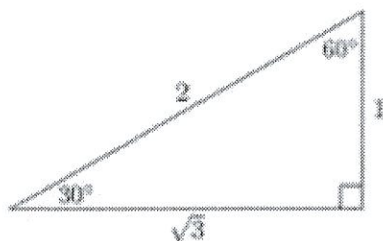


Name:

Solutions / Answers

Directions: All fractions must be reduced (simplified fully). Denominators may contain radicals.

1. Using the special triangles shown, Evaluate each trigonometric ratio:



a. (2 pts) $\sin 45^\circ = \frac{\sqrt{2}}{2}$
 $\frac{1}{\sqrt{2}}$

b. (2 pts) $\tan \frac{\pi}{3} = \sqrt{3}$

c. (2 pts) $\sec 45^\circ = \sqrt{2}$
 $\frac{1}{\cos 45^\circ} = \frac{1}{(\frac{1}{\sqrt{2}})}$

d. (2 pts) $\csc \frac{\pi}{6} = 2$
 $\frac{1}{\sin \frac{\pi}{6}} = \frac{1}{(\frac{1}{2})}$

e. (2 pts) $\cos 60^\circ = \frac{1}{2}$

f. (2 pts) $\cot \frac{\pi}{6} = \sqrt{3}$
 $\frac{1}{\tan \frac{\pi}{6}} = \frac{1}{(\frac{1}{\sqrt{3}})}$

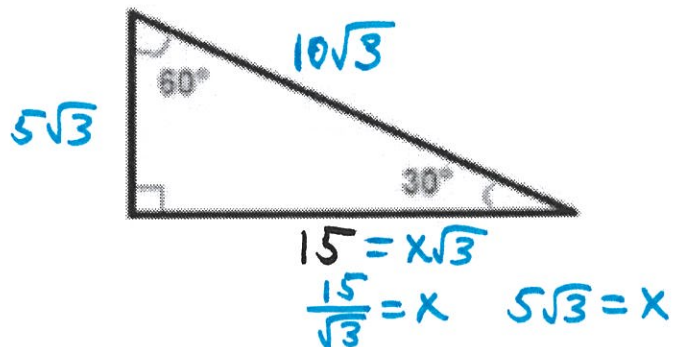
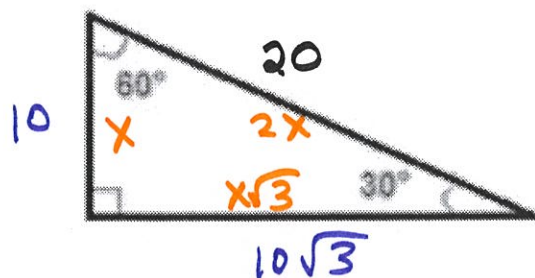
g. (2 pts) $\sin 60^\circ = \frac{\sqrt{3}}{2}$

h. (2 pts) $\tan \frac{\pi}{4} = 1$

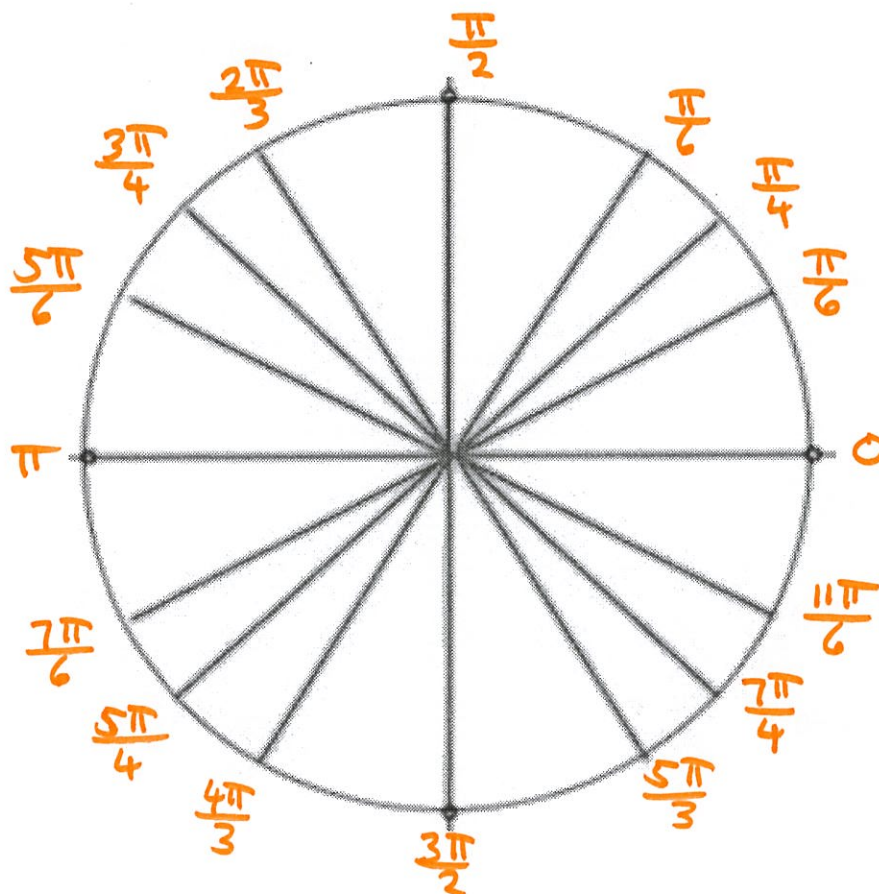
i. (2 pts) $\csc 45^\circ = \sqrt{2}$
 $\frac{1}{\sin 45^\circ} = \frac{1}{(\frac{1}{\sqrt{2}})}$

j. (2 pts) $\cot \frac{\pi}{3} = \frac{\sqrt{3}}{3}$
 $\frac{1}{\tan \frac{\pi}{3}} = \frac{1}{\sqrt{3}}$

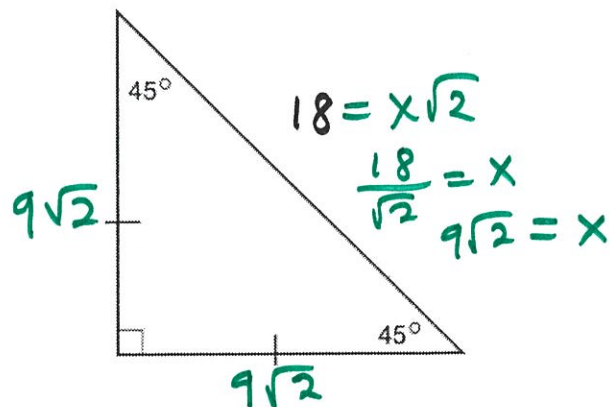
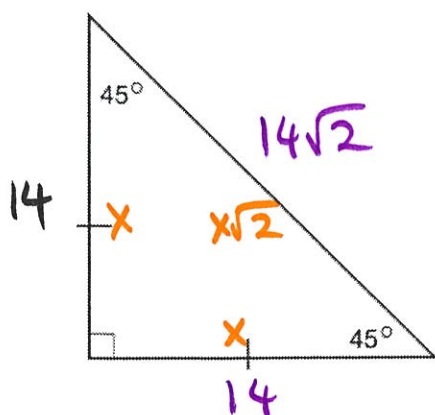
2. For each triangle, determine the length of the sides whose lengths are not given (2 points per side length).



3. Label all 16 special angle RADIAN measures within one rotation of a circle. Give reduced fractions. (2 points per angle).



4. For each triangle, determine the length of the sides whose lengths are not given (2 points per side length)



5. Convert each angle measure from degrees to radians. Give reduced fractions (2 points per angle).

a) $120^\circ = \frac{2\pi}{3}$	b) $225^\circ = \frac{5\pi}{4}$
c) $330^\circ = \frac{11\pi}{6}$	d) $270^\circ = \frac{3\pi}{2}$

6. Convert each angle measure from radians to degrees (2 points per angle).

a) $\frac{3\pi}{4} = 135^\circ$	b) $\frac{4\pi}{3} = 240^\circ$
c) $\frac{11\pi}{6} = 330^\circ$	d) $\frac{\pi}{2} = 90^\circ$

1. (2 points) Write the coordinates on the unit circle at the $\frac{3\pi}{4}$ angle point.

$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

2. (2 points) Write the coordinates on the unit circle at the π angle point.

$$(-1, 0)$$

3. (2 points) Write the coordinates on the unit circle at the $\frac{7\pi}{6}$ angle point.

$$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

4. (2 points) Write the coordinates on the unit circle at the $\frac{4\pi}{3}$ angle point.

$$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

5. (2 points) Write the coordinates on the unit circle at the $\frac{3\pi}{2}$ angle point.

$$(0, -1)$$

6. (2 points) Evaluate the trigonometric expression $\tan \frac{2\pi}{3} = -\sqrt{3}$

7. (2 points) Evaluate the trigonometric expression $\tan \frac{\pi}{2} = \frac{1}{0} = \text{undefined}$

8. (2 points) Evaluate the trigonometric expression $\sin \frac{5\pi}{6} = \frac{1}{2}$

9. (2 points) Evaluate the trigonometric expression $\csc \pi = \frac{1}{\sin \pi} = \frac{1}{0} = \text{undefined}$

10. (2 points) Evaluate the trigonometric expression $\cot 225^\circ = \frac{1}{\tan 225^\circ} = \frac{1}{1} = 1$

11. (2 points) Evaluate the trigonometric expression $\sec \frac{11\pi}{6} = \frac{1}{\cos \frac{11\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

12. (2 points) Evaluate the inverse trigonometric expression $\sin^{-1}(0) = 0^\circ \text{ or } 0$

13. (2 points) Evaluate the inverse trigonometric expression $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \text{ or } 30^\circ$
 $\sin \theta = \frac{1}{2}$

14. (2 points) Evaluate the inverse trigonometric expression $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \text{ or } 30^\circ$

15. (2 points) Evaluate the inverse trigonometric expression $\cos^{-1}(1) = 0^\circ \text{ or } 0$

16. (2 points) Evaluate the inverse trigonometric expression $\tan^{-1}(-1) = -\frac{\pi}{4} \text{ or } -45^\circ$

17. (2 points) Evaluate the inverse trigonometric expression $\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3} \text{ or } 120^\circ$

18. (2 points) Evaluate the inverse trigonometric expression $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{3} \text{ or } -60^\circ$